Rational Function Model Optimization Based On Swarm Intelligence Metaheuristic Algorithms

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Abstract. The Rrational Function Model (RFM) is progressively being familiar to the mapping and photogrammetric researchersand has been widely used as an approximate to rigorous Models. According to it ability to preserve the complete accuracy of various types of physical sensors and its independence of sensors and platforms, it can be had with any coordination system. Nevertheless, the RFM coefficients are also known as rational polynomial coefficients (RPCs) dependent on a large number of ground control points which makes the model susceptible to over parameterization error and a time-consuming also the RPCs have no physical meaning, as a result, selecting the best combination of RPCs is difficult. The intelligent algorithms based meta-heuristic optimization seem to be an effective approach for overcoming this problem. This paper focuses on the application of recent swarm intelligence based meta-heuristic algorithms for RFM optimization. The most popular optimization methods considered are ant colony algorithm, genetic algorithms and particle swarm optimization.Furthermore in this research we proposed an parallel hybrid meta-heuristic optimization algorithm that combines the genetic algorithm and particle swarm optimization concepts to overcoming the swarm intelligent limitations for RFM optimization. The different algorithms are applied for two data sets provided from the Algerian satellite (ALSAT2).The results demonstrated that the proposed method is more accurate than the threesuggested based meta-heuristic methods.

Keywords: Rational function model, ant colony algothms, Genetic algorithms, particle swarm optimization, hybrid algorithms.

1 Introduction

One of the most important sources of geographic information systems (GIS) is high-resolution satellite imagerythat have been used in many applications such as remote sensingand topograghic maps.But Raw images usually contain some significant geometrical distortions which cannot used directly in GIS without ortho-rectification. Ortho-rectification is a standard process for correcting the geometric distortions and relief displacement errors introduced during acquisition [1,2]. The high accuracy potential of ortho-rectification depends on the relationship between images and object spaces [3]. For this end, a wide variety of mathematical models have been developed which generally fall into two main physical (rigorous or sensor dependent) and non-physical (empirical or sensor independent) models [4].The most frequently used non-parametric model are based on 3D rational polynomials and which in literature are known as the Rational Function Model(RFM). Unlike rigorous models, RFM does not need any physical understanding of the sensor or satellite. RFM has received increasingly more acceptance because of its simple form, convenience for using, low requirement for specialized knowledge and not depending on the imaging parameters of specific satellite.

There are two methods to solving the RFM named as dependent-terrain and independent-terrain. In the case of the independent terrain, RFM is solved by using the physical sensor model which requires the availability of some information on the sensor (attitude and orbital parameters). In the independent-terrain approach, RFM necessitates a large number of accurate, well-distributed ground control points (GCPs) which is a time-consuming and costly process. In addition, RFM coefficients or also rational polynomial coefficients (RPC) have no physical significance which makes it difficult to find their best combination [5]. To overcome these problems the binary form of meta-heuristic algorithms can be helpful in optimization and determining the optimum RPCs.

Nowadays, computational intelligence and meta-heuristic algorithms have become increasingly popular in computer science, artificial intelligence, machine learning, image processing and data mining. Most algorithms in computational intelligence and optimization are based on swarm intelligence (SI) [6] that explores the behavior of natural entities (consisting of many individuals) in order to build artificial systems for solving problems of practical relevance. Therefore, among population-based we distinguish SI-based metaheuristics such as Ant Colony Optimization (ACO), genetic Algorithms (GA), and Particle Swarm Optimization (PSO) which are the most popular optimization algorithms that often have excellent ability for self-organization, self-learning, or self-memory [7]. A few studies have been conducted on the use of artificial intelligence algorithms to solve terrain-dependent RFMs such as GA in [8–10] and PSO [11,12].

In this paper, we examines three algorithms: ant colony optimization (ACO) , genetic algorithm (GA) and particle swarm optimization adaptive to RFM optimization (PSORFO). In second part we have proposed a parallel hybrid meta-heuristics that combine the GA and PSORFO concepts to overcoming their limitations and avoid the local minima.

2 *Rational Function Model (RFM)*

RFM is is a mathematical model that uses a ratio of two polynomial functions to define the relationship between a point in-ground space(X,Y,Z) and it corresponding in image space (r,c)or vice versaas indicated in the following equation [13] :

$$r = \frac{P_1(X,Y,Z)}{P_2(X,Y,Z)},\ c = \frac{P_3(X,Y,Z)}{P_4(X,Y,Z)} \tag{1}$$

the3D polynomial function P_i (i=1,2,3,4) is defined as:

$$P_n = \sum_{i=0}^{m1} \sum_{j=0}^{m2} \sum_{k=0}^{m3} a_{ijk} X^i Y^j Z^k \tag{2}$$

where :

$$\begin{gathered} n = 1,2,3,4 \\ 0 \le m1 \le 3 \\ 0 \le m2 \le 3 \\ 0 \le m3 \le 3 \\ m1 + m2 + m3 \le 3 \end{gathered}$$

The a_{ijk}indicate the coefficients of the RFM and are referred to as Rational Polynomial Coefficients (RPC), or Rational Function Coefficients,(RFC) [1, 13], to detremines the RPCs values we use a set of *ground control points* (GCP) for which the r,c,and X,Y,Z coordinates are known, must be taken into account, the first coefficient in the denominators(P2 and P4) are supposed to be 1, resulting there are 39 RPCs in each equation: 20 in the numerator and 19 (with the constant 1) in the denominator. So a minimum of 39 GCPs are needed to solve the 78 coefficients.It is necessary to consider errors in the reference points, which include not only GCPs but also check points, in estimating the accuracy of the results [11].

The linearized RFM type can be used to solve unknown RPCs [13], as seen below:

$$P_1(X,Y,Z) - rP_2(X,Y,Z) = 0 \tag{3}$$

$$P_3(X,Y,Z) - cP_4(X,Y,Z) = 0 \tag{4}$$

Using n number of GCPs, the above equations can then be written as follows:

$$y = Ax + e \tag{5}$$

where :
A: design matrix
y: observations vector
e: residuals vector
x: vector of RPCs.

RPCs can be determined using the least-squares (LS) method, as shown below:

$$x = (A^T A)^{-1} A^T y \tag{6}$$

3 Swarm Intelligence Based Meta-heuristic Algorithms Applied for RFM Optimization Review

Several algorithms have been suggested in the literature, most of which are based on natural phenomena. Among them, certain swarm intelegent meta-heuristic search algorithms built on a population-based structure have proven to be effective at solving optimization problems. The algorithms : Ant colony optimization (ACO), genetic algorithm (GA), and particle swarm optimization (PSO) are the most popular belong to this category. These algorithms, as well as their improved scheme for RFM optimization have demonstrated in this section.

3.1 Binary Ant Colony Optimization (BACO)

The ant colony optimization algorithm was inspired by the walk of ants on the search for food [14]. The binary version of ant colony algorithm (BACO) is apply for RFM optimization, in this section the BACO is described, the algorithm starts by creating the environment in which our ant-like agents will evolve [15,16]. Fig.1 illustrates this environment, which is composed of two lines (sequences of 0 and 1). The ants start the search process from the root, as is described in the following figure.

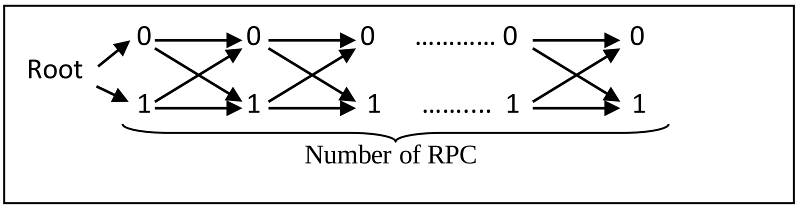

Fig.1. Ant colony search space

At the t[th] iteration, N ants produce a solutions in the form of binary sequences by come in the field from the left and exits from the other side, as a result, each ant formed while crossing the field a solution (RFM structure) , the length of the field equal the dimension of RFM structure (number of RPC) , the artificial ant (k) at each node chooses either 0 or 1 as the next node to walk according to the pheromone vector $T(t)^k = \tau_{1x}^k(t) + \tau_{ix}^k(t) + \cdots + \tau_{nx}^k(t)$, where τ_{ix}^k is the pheromone value for x which is a binary number (0,1) from the sequence i=1,…,n of the k[th] ant . The pheromones are updated regularly during the search, and initially set to value (τ_{init}) at the start of the search. An ant chooses which way to go when visiting a node (0 or 1) by based to the transformation probabilities p [16] **:**

$$x_{0,1}(t) = \frac{(\tau_{0,1}(t))^{\alpha}}{\sum_{l\epsilon N_i^k}(\tau_{0,1}(t))^{\alpha}} \tag{7}$$

$$p_{0,1}^k(t) = \frac{(x_{0,1}(t))}{\sum_{l\epsilon N_i^k}(x_{0,1}(t))} \tag{8}$$

Where the parameter αcontrols the relative weight of pheromone trail in the probabilitycomputation.The ant then chooses the next step based on the probability p, repeating the procedure until it amount to the last bit, the process continues until N ants have finished their walk across the field, as a consequence, N solutions are produced.

The binary string discovered during the N ants walk through the field is considered as solutions to the problem (RFM optimization) and assessed by the fitness function. After that, the quantity of pheromone at all connections is evaporated using (Eq. 9), where the initial evaporation rate set to ρ value.

$$\tau_{ix}(\mathrm{t}+1) = (1-\rho)\tau_{ix}(\mathrm{t}) \tag{9}$$

Finally the pheromone reinforces at the connection with value proportional to each solution as in (Eq.10) whichis bounded between pheromone minimum τ_{min} and pheromone maximumτ_{max}

$$\tau^{k}_{0,1}(\mathrm{t}) = \tau^{k}_{0,1}(\mathrm{t}) + \frac{1}{(1+\mathrm{e}^{(\text{fitness value})^{\mathrm{k}}})^2} \tag{10}$$

3.2 Genetic Algorithms

The Genetic algorithms (GAs) are heuristic search optimization algorithms developed firstly by J. Holland [17] based on the concept of natural selection and genetics inspired by Darwin's theory. They represent an intelligent exploitation of a random search that combine both exploration and exploitation in an optimal way. Owing these interesting properties, GAs presents a robust technique that can deal successfully with a wide range of problem areas including applied in remote sensing and especially for RFM optimization [8,9].

GAs are an iterative process that evolves under a population of individuals or chromosomes generated randomly which represents a potential solutions of the problem. In computing, chromosomes can be represented by strings. The most commonly used alphabet of the strings is binary, but other alphabets are also used, e.g, integer or real valued numbers, depending on which is the most suitable for a given problem.

Each individual is a candidate solution to the optimization problem which is assigned a fitness value based on the objective function of the problem. In the selection step, the best fittest of the individuals are chosen and are more likely to survive in the next generation. Therefore, the mechanism of reproduction involves crossover and mutation operations in order to generate a new population of individuals. The purpose of those operations is to modify the chosen solutions and select the most appropriate offspring to pass on the succeeding generation until no better fitted solutions are possible. The process is iterated from generation to another until a defined termination criterion has been reached as like the number of generations or a satisfactory fitness level.

3.3 Binary Particle Swarm Optimization

Particle swarm optimization (PSO) is one of the most common meta-heuristic optimization algorithms based on social intelligence and cooperative behavior shown by a swarm of birds or fish while searching for food. The first version of the particle swarm algorithm was developed by James Kennedy and Russell Eberhart in 1995, works in continuous search space [18]. Application of the standard PSO is not complicated, the algorithm is characterized by two fundamental parameters the position (location) and velocity of each particle. Where the velocity of the particle i is bounded between velocity minimum and velocity maximum [Vmin, Vmax], and calculating at iteration t (starting with 0) by the following equation[19,20]:

$$\mathrm{v_i}(\mathrm{t}+1) = \mathrm{w(t)v_i(t)} + \mathrm{c_1 r_1}\big(\mathrm{p_i(t)} - \mathrm{x_i(t)}\big) + \mathrm{c_2\, r_2}\left(\mathrm{p_g(t)} - \mathrm{x_i(t)}\right) \tag{11}$$

Where w is time-varying inertia weight, for RFM optimization w(t) is a decreasing function of iterations, as to eq.12, which is used in [20].

$$w(t) = w_{min} + (w_{max} + w_{min}) \,.\, \frac{t_{max} - t}{t} \quad (12)$$

t_{max} is the maximum number of iterations , w_{min} and w_{max} are two constants experimental parameters , *c1* and *c2* signify the acceleration factors. Generally *c1* equals *c2*, *r1* and *r2* are two random numbers within a range of [0,1], P_g denotes the best particle of the swarm giving the best objective function value (best solution), and the best previous position of the i^{th} particle is represented as P_i *and x* is the present position (solution) of the particle *i*.When the velocity is determined, the position of the particle *i* is updating from:

$$x_i\,(t+1) = x_i\,(t) + v_i\,(t+1) \quad (13)$$

PSO was used in many fields of research and application but certain problems of optimization are solved in discrete space, not in continuous search space. It is for this reason that Kennedy suggested a binary (discrete) version of the particle swarm optimization (BPSO) in [20]. The algorithm proposes to make the velocity of particle i as an input of sigmoid function to obtain the value 0 or 1 for the position of the particle i and the position is updating as follows:

$$x_i\,(t+1) = \begin{cases} 1, & \text{if } r_i < S(v_i(t+1)) \\ 0, & \text{otherwise} \end{cases} \quad (14)$$

$$S(v_i(t+1)) = \frac{1}{1+e^{-v_i(t+1)}} \quad (15)$$

Where velocity is updated with the same equation (eq.12). BPSO has proven to be effective in RFM optimization in many research studies. Yavari proposed a modified version of PSO adaptive to RFM optimization named in this work PSO-RFO in [11] by using a novel normalization function as a substitute for the sigmoid function(eq.16 and eq.17)

$$x_i\,(t+1) = \begin{cases} 1, & \text{if } r_i < \phi(v_i(t+1)) \\ 0, & \text{otherwise} \end{cases} \quad (16)$$

$$\phi(v_i(t+1)) = \begin{cases} \tanh\big(v_i(t+1)\big), & \text{if } v_i(t+1) > 0 \\ 0, & \text{otherwise} \end{cases} \quad (17)$$

3.4 The proposed Paarallel Hybrid GA-PSO Algorithm for RFM Optimization

PSO and GAs have a lot of similarities in their characteristics, but studies demonstrate that they each have their limitation for solving various problems [21] in order to maximize and combine their strengths while overcoming their weaknesses for RFM optimization. This study proposes a parallel hybrid approach that combines the concepts of the binary version of GA and PSORFO named PHGA-PSO.

The different steps of the proposed algorithm are summarized as follows :

- Step 1. (Initialization) :Individuals are randomly generated. In the case of PSO, these individuals are particles, and in the case of GAs, they are chromosomes.
- Step2: Calculate the cost value of all individuals in population; the population is then grouped into two subgroups of equivalent individuals based on the cost value computed.
- Step3: From a total of N individuals, the first N/2 are selected as subgroup to apply the GA steps while the bottom 2N are chosen to form a subgroup for PSORFO steps, The obtained cost values are compared to determine the global best value (optimum value).
- Step4. (Termination criteria): The Steps 2–3 will be repeated until the current iteration achieves the maximum number of iterations.

Figure 2 describes the proposed algorithm PHGAPSO for RFM optimization.

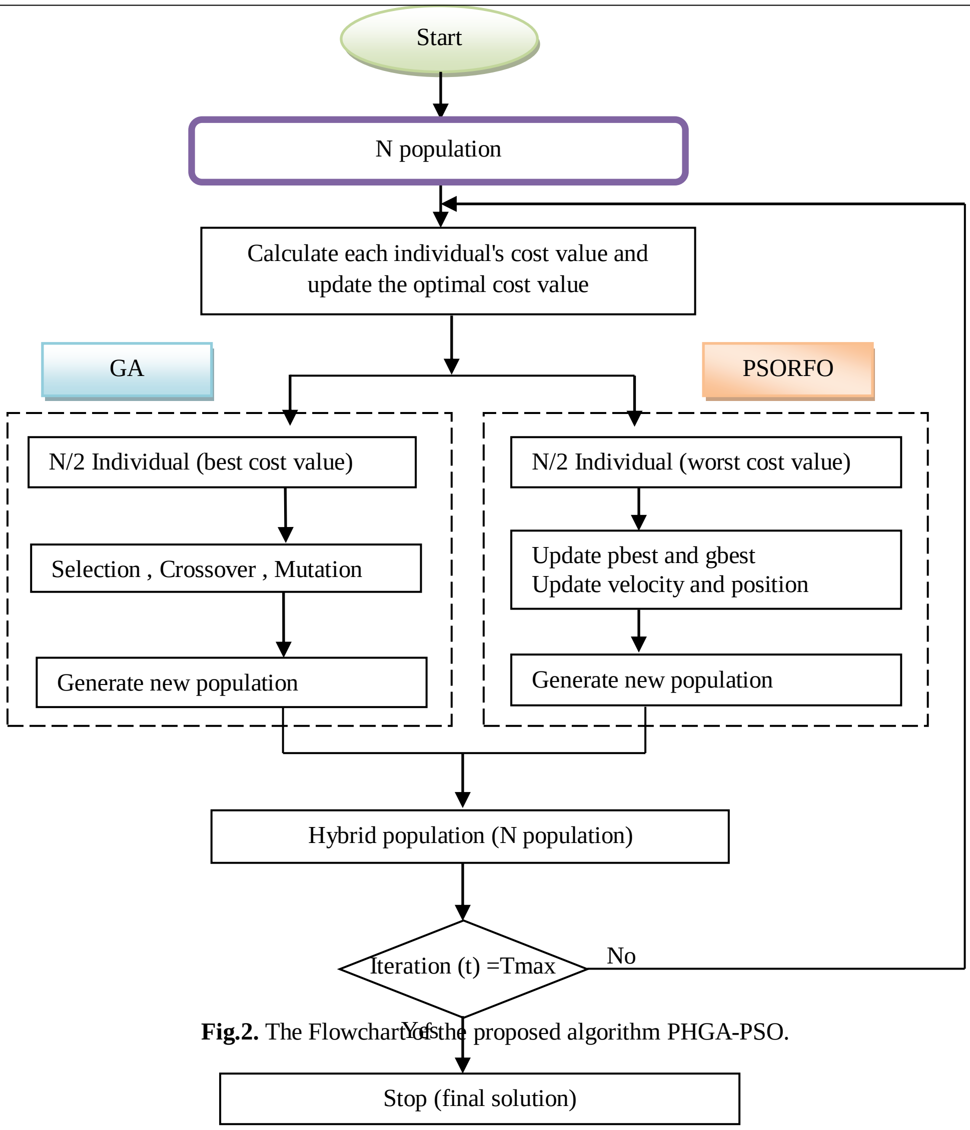

Fig.2. The Flowchart of the proposed algorithm PHGA-PSO.

4 Experimental Tests and Discussion

In this paper, we used two data sets for experimentat tests provided by the first Algerian high-resolution satellite ALSAT2 taken over different cities in Algeria. The first data is a multispectral image over the region of Algiers acquired in September 2014 with a dimension of 1750×1750 pixels and a total of 20 control points. The second one is over the region of Oran acquired in June 2016 of size 3500×1750 pixels and 18 control pointst. The distribution of the GCPs over this regions were acquired using geodetic survey techniques, the number and the distribution were optimized for ortho-rectification with rigorous model in a real production.

The experiments in this study were performed with a MATLAB code and executed in a MATLAB R2017a. All the tests were compiler on a personal computer, Intel Core i3 CPU 2.40Ghz with an 8.00 GB available RAM, we declared the maximum number of iterations (Tmax) to be a termination condition and is set to 200. RFM version used in this work has 78 parameters (78 RPCs) which it is often used in remote sensing.Thus each solution is represented by a string of 78 binary values;where a “one” indicates the presence of the corresponding RPC coefficient in RFM and a “zero” indicates its absence. The population size is fixed at 30for all tested methods. Table 1 depicts the remaining parameters used for each method.

The processus of RFM optimization is made by using a set of control points which is pratically divided into three types of groups:

1. Ground control points (GCPs) used in Least Square method for determine the RPCs.
2. Dependent Checkpoints (DCPs) used to calculate the fitness value.
3. Independent Checkpoints (ICPs) used to assess the whole accuracy of the method.

So, a combinations of well-distributed GCPs are used for determining RPCs coefficients, 20 % of GCPs were assigned as DCPs and a set of ICPs are used to evaluate the accuracy of the algorithm.The most popular metric used in photogrammetry is the Root Mean Square Error (RMSE) that is used as a cost function for DCP and accuracy assessment to ICP given by this equation:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(x_i-\hat{x}_i)^2+(y_i-\hat{y}_i)^2}{N}} \quad (18)$$

Where: N is the total number of DCPs, x_i, y_i the estimated coordinate(x, y), $\hat{x}_i, \hat{y}_i$:the actual coordinate (x, y).

Table 1. Parameters of the tested methods.

ACO		GA		PSO		
τ_{init}	0.5	Crossover probability	0.075	Velocity	v_{min} v_{min}	-3 +3
τ_{min}	0.05	Mutation probability	0.001	Inertia weight	w_{min} w_{max}	0.02 1
τ_{max}	0.95			Acceleration factors	C1 C2	0.5 0.5
α	1					
ρ	0.0004					

The experiments have been divided into two sections, the first is a comparison between tested methods (ACO, GA, PSORFO, PHGA-PSO) in terms of accuracy, the second experiment is in term of convergence speed and computation time.

❖ **The accuracy Test**

The quality assessment of the results is performed with different combination of GCPs and the RMSE function is the evaluating metric. RMSE is calculated over ICPs to determine the accuracy of the obtained results. As every execution of the meta-heuristic algorithms produced a different result, the algorithms were executed 10 times; the best one (with the lowest cost function) among the ten was chosen for accuracy test.

Table 2. Accuracy results of the tested algorithms.

Data	GCP/ICP	RMSE over ICP			
		BACO	GA	PSORFO	PHGA-PSO
Algiers	14/6	1.7539	1.8358	1.2378	1.0574
	12/8	4.9463	2.2168	1.2532	1.5275
	9/11	543.6984	21.4128	2.5673	2.0290
Oran	13/5	55.8999	30.6274	4.4242	6.0541
	10/8	994.9687	39.6274	10.4139	6.5261
	9/7	1.9133e+03	2.1077e+03	10.6425	10.3282

As seen in Table 2, the PHGA-PSO outperforms the other methods in most cases, the RMSE value demonstrates the high accuracy of the proposed method this is due to mixed of GA concept with PSORFO which giving more variety in solution compared to other tested algorithms.

For the first data set (Algiers), when compared the tested literature methods in term of accuracy the BACO and GA have a low efficiency with limited number of GCP (case of 9 points) in other hand the PSORFO and PHGA-PSO, showed accurate results equal to 2.567

and 2.029 pixel respectively , for the cases of 12 and 14 GCPs , all methods produced proper results.

In the second data set (Oran), the obtained results obviously demonstrate a decrease in accuracy, which can be interpreted by the ground points distribution over this dataset, where the points are distributed for a real production case using a rigorous model, it can also be explained by the size of the image, for PSORFO and PHGA-PSO, the results are satisfactory if we compared with the size of image and the number of ground points, however, and BACO didn't get a good results and be affected to the image length was twice as long as the first data set, in return a smaller number of points.

The overall analysis of Average RMSE values indicates that the PHGA-PSO results are on average better in accuracy compared to other tested method, as a result, we can declare that the PHGA-PSO remains the best method in term of accuracy, especially when there are a limited number of ground points.

❖ **Convergence speed and computation time:**

In this section the average execution times of tested literature methods on the experimental data sets are studied. Figure 3 demonstrates the average performance time in second (s) of the proposed method and the other methods on the two data sets. In two data sets, the computational time of PHGA-PSO was significantly slower than that of other methods: GA, PSORFO and BACO, this is due to the fact that PHGA-PSO contains much operation because it mixes the GA and PSO, which significantly increases the processing time. we notice also that the PHGA-PSO ,PSO and BACO take less time to compute than GA, this is because GA comprises a complex operations (selection, crossover, mutation).

The best run among the ten runs is chosen for convergence speed analysis in order to evaluate the convergence of the literature methods, the convergence curve of the literature methods with a set of 14 GCPs for Algiers data set is seen in Fig.4, we choose this case because it produces the best results among the others. The BACO and GA have a faster convergence rate than PSORFO and PHGA-PSO, implying that the BACO and GA need less number of iterations than PSORFO and PHGA-PSO, for GA requiring approximately less than 20 iterations and BACO requiring less than 40 iterations.

We may summarize the findings results as follows: the GA is more time consuming and faster convergence than the other algorithms tested, in other side the PSORFO is the fastest in terms of average processing times. this is due to the structure of the algorithm, which the PSORFO is the simplest.

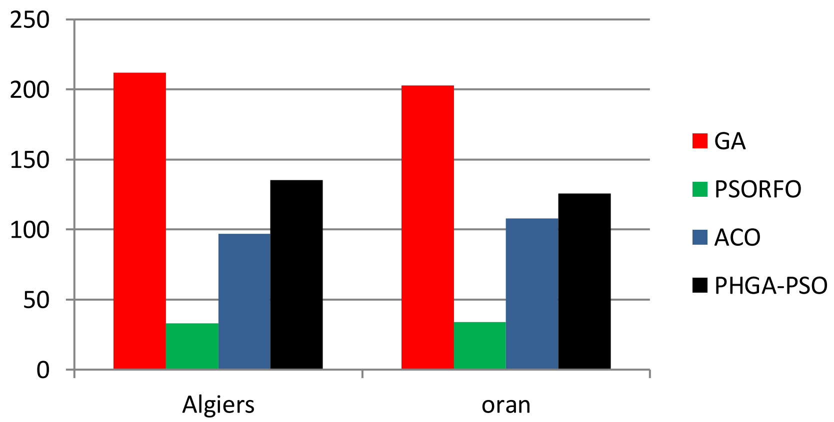

Fig.3. Computational times of the tested algorithms.

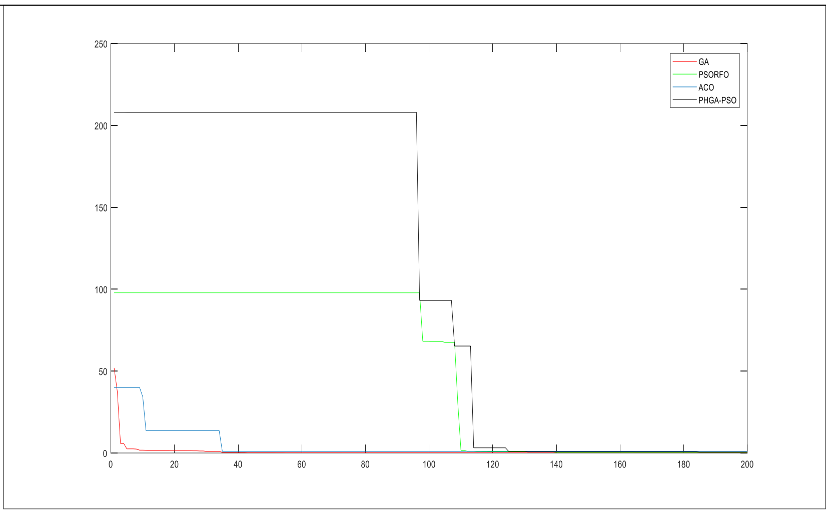

Fig.4. Convergence accuracy over iterations of Algiers data set.

5. Conclusion

The paper discusses the use of certain meta-heuristic algorithms, such as ant colony optimization (ACO), genetic algorithm (GA), and particle swarm optimization (PSO) for RFM optimization and to solve the over parameterization problem owing due to the significant number of RPCs existing in RFM. GA and PSO have demonstrate their superiority than ACO for RFM optimization but each algorithms GA or PSO have their limitation for solving the over parameterization problem existing in RFM, in order to combine their advantages while overcoming their limitations , in this paper we proposed a novel parallel hybrid meta-heuristic optimization algorithm (PHGA-PSO), it combines the genetic algorithm (GA) and particle swarm optimization (PSO) concepts, the PHGA-PSO utilizes a combination of selection, crossover, and mutation from genetic algorithms (GAs) with the update velocity and position from particle swarm optimization (PSO), all while maintaining PSO and GA independence by divided the algorithm in subgroup group for GA and group for PSO.

These tested literature methods applied for different size of satellite images provided by Algerian satellite (ALSAT2) .The experimental results demonstrate that the proposed PHGA-PSO is advantageous to other algorithms in accuracy term and finding the best RPCs combination.

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