Approximate method for oxygen diffusion and absorption in sick cell

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Abstract

we consider the oxygen diffusion problem where the injection of oxygen into a sick cell and diffusion of the injected oxygen inside the cell. The problem mathematically formulated through two different steps. At the first stage, the stable case having no oxygen transition in the isolated cell is searched while at the second stage the moving boundary of oxygen absorbed by the tissues in the cell is searched. In this study, trace of moving boundary of the oxygen diffusion problem is determined using constrained integral method, the profile of moving boundary is determined by third order polynomial.

Keywords : Oxygen diffusion; Constrained integral method; Moving boundary problem; Stefan problem.
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1 Problem

The moving boundary problem arising in biomechanical diffusion theory which is formulated in Seval Çatal [4](2003). This type of problem was studied by Crank and Several authors [1], [2]. We see that the analytical solution is difficult to obtain and the moving boundary is an essential peculiarity of this problem. The oxygen diffusion in sick cell is generally presented in two stage. First oxygen is allowed to diffuse into a sick cell.The second stage is that of tracing the movement of the boundary and determining the distribution of the oxygen in the cell. We express in non-dimensional form, the problem is giving by (Seval Çatal 145 (2003) 361 − 369)[4] is:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - 1 \quad 0 \leq x \leq s(t) \tag{1.1}$$

the boundary condition

$$\frac{\partial u}{\partial x} = 0 \quad \text{at the sealed surface} \quad x = 0 \quad t \geq 0 \tag{1.2}$$

$$u = \frac{\partial u}{\partial x} = 0 \quad \text{at the moving boundary} \quad x = s(t) \quad t \geq 0 \tag{1.3}$$

and the initial conditions at $t = 0$ are

$$u = \frac{1}{2}(1 - x)^2 \quad 0 \leq x \leq 1 \tag{1.4}$$

with

$$u = 0 \quad , \quad x = s(t) = 1 \quad \text{and} \quad t \geq 0 \tag{1.5}$$

We note that $u(x, t)$ is the concentration of oxygen free to diffuse at a point x, at time t and the location of the moving boundary is $s(t)$.

References

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